Dynamical systems

Dynamical Representation

- Dynamical systems originated (Newton, Leibnitz) to describe the motion of objects.
- But today, they are widely employed to understand (and model) an enormous variety of phenomena.
  - population growth and decay
  - weather
  - neural changes during learning
  - locomotion
  - chemical reactions
  - economic growth and decline
  - brain circuits

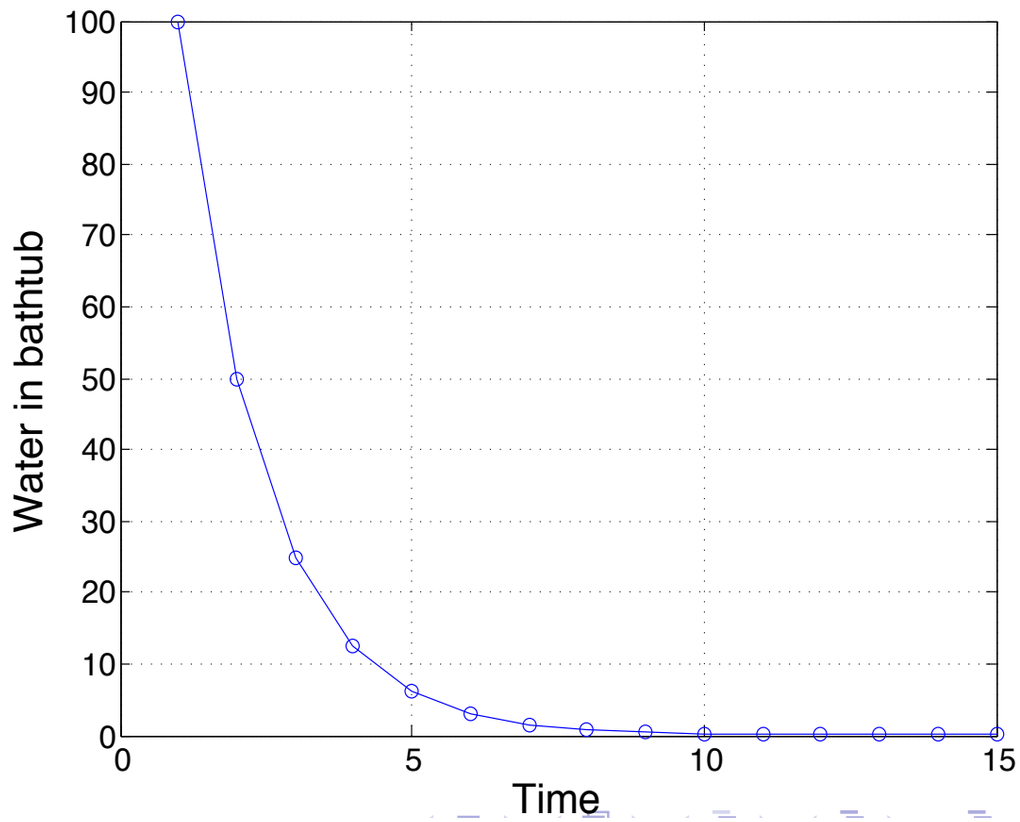
What is a dynamical system?

- Equation (or set of equations) that quantitatively describes the change in something over time or space.
- Two parts:
  1. State of the system (quantity whose change is being described)
     - amount of money in the bank
     - number of individuals in a population
     - volume of water in a bathtub
     - activation level of a neuron
     - the concentration of Sodium in a solution
     - the speed of traffic on the 101
     - the temperature in a room
  2. Rule for how the state changes, depending on the current state. Current state predicts the state at the next instant in time.
- System is constant, even while state is changing.

Example Dynamical System

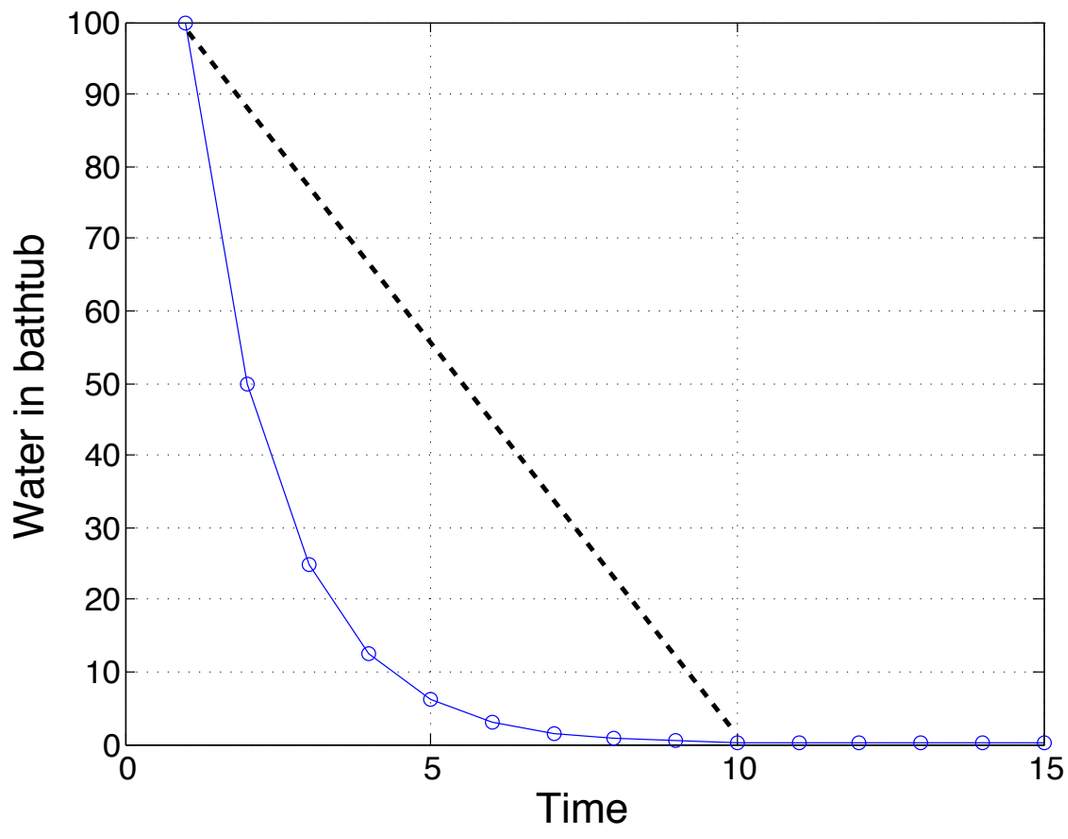
- State: Amount of water in bathtub (x)
- Rule for change:
  Change in x $= -\frac{1}{2}x$

Time	H_20
1	100
2	50
3	25
4	12.5
5	6.25



Example Dynamical System

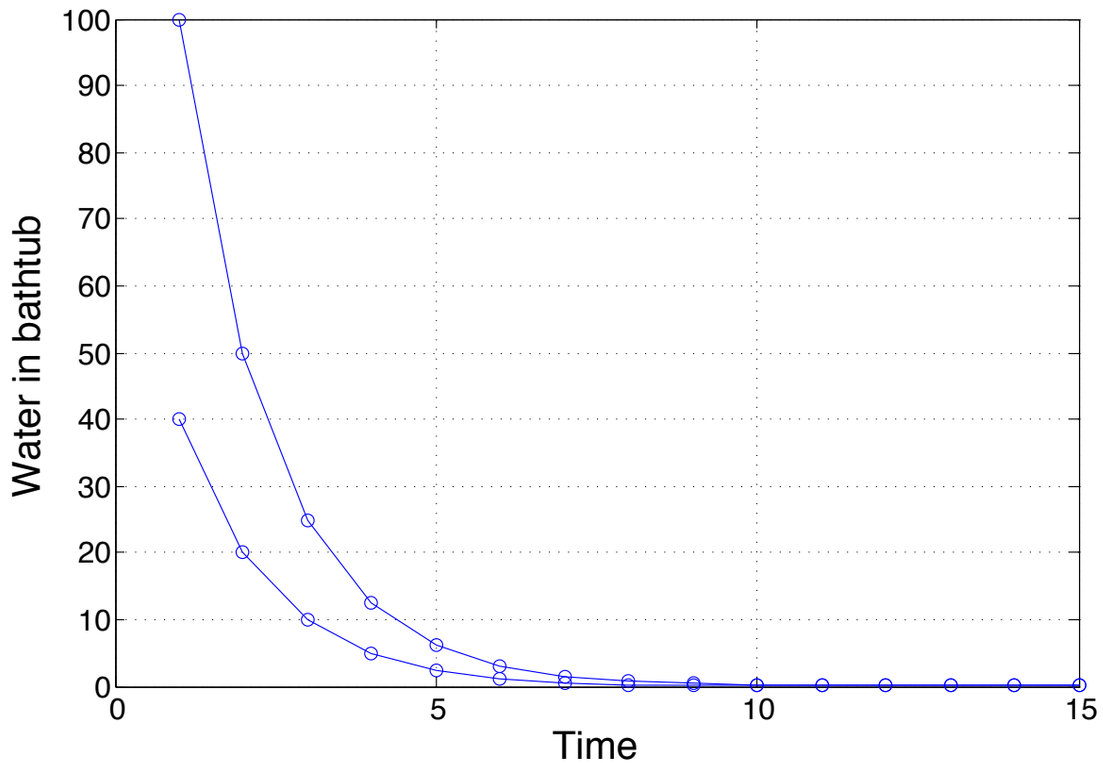
- Alternative rule for change:
- Rule for change:
  Change in x $= -10$



How do you know that this isn't right for emptying bathtub?

Initial Conditions

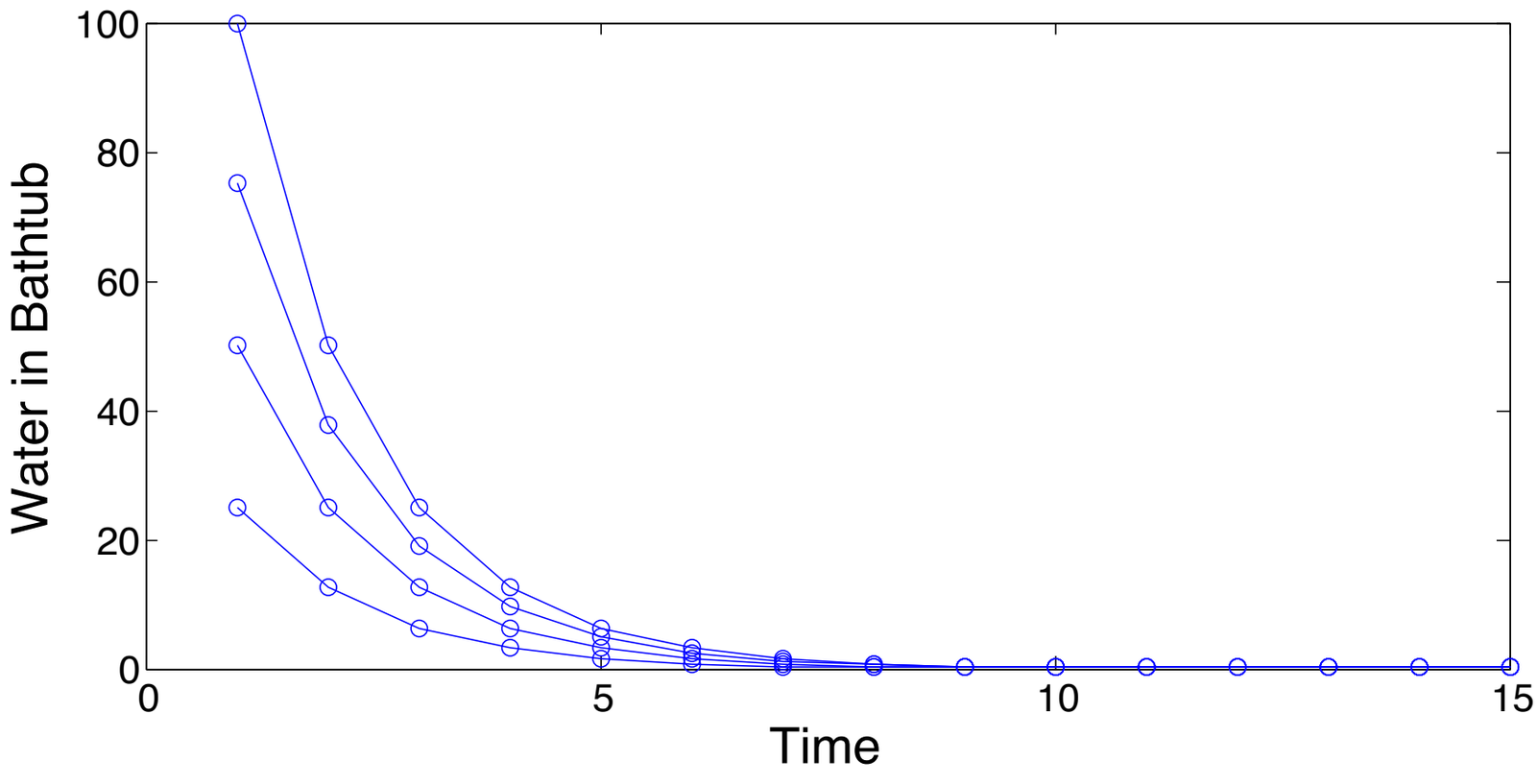
- Suppose the bathtub has 40 units of water at time 1.
- Same rule for change:
  Change in x $= -\frac{1}{2}x$



Same rule for change produces very different functions over time.

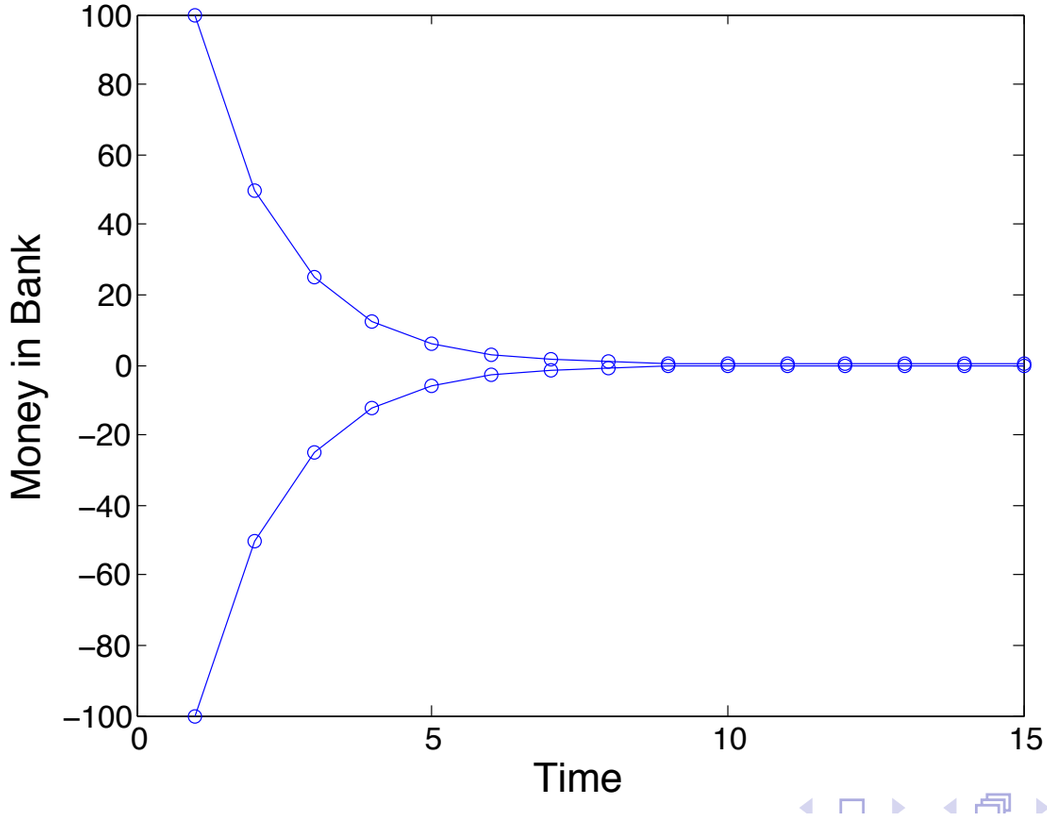
Goal or Point Attractor

- With this rule for change, all initial conditions wind up at 0.
- We can call this a *goal* of this system.



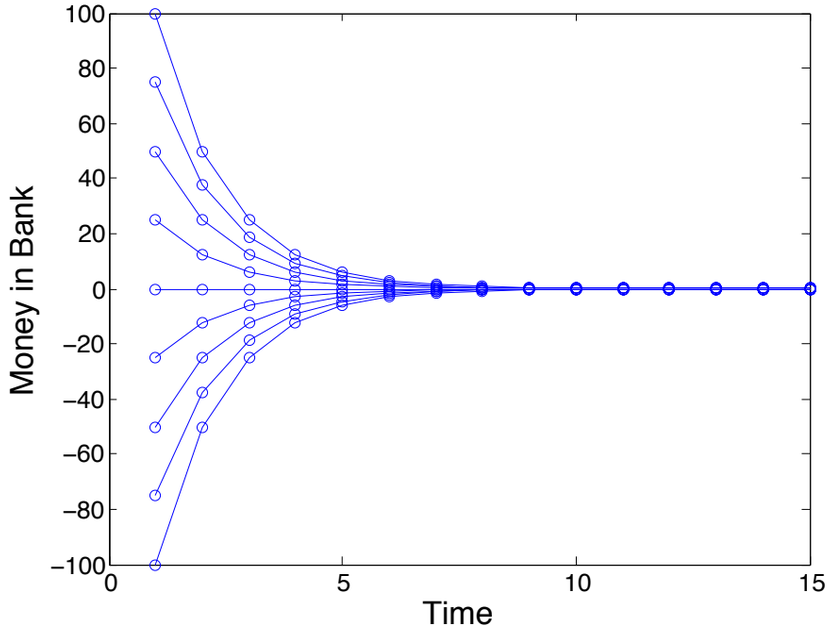
More initial Conditions

- Same rule for change:
  Change in x $= -\frac{1}{2}x$
- Suppose the state being modeled is the amount of money in your bank account.
- Balance can be positive or negative
- Suppose you begin with a negative balance of \$100



Goal

- Same rule for change:
  Change in x $= -\frac{1}{2}x$
- All initial conditions, positive, negative, zero, result in the same goal value, 0.

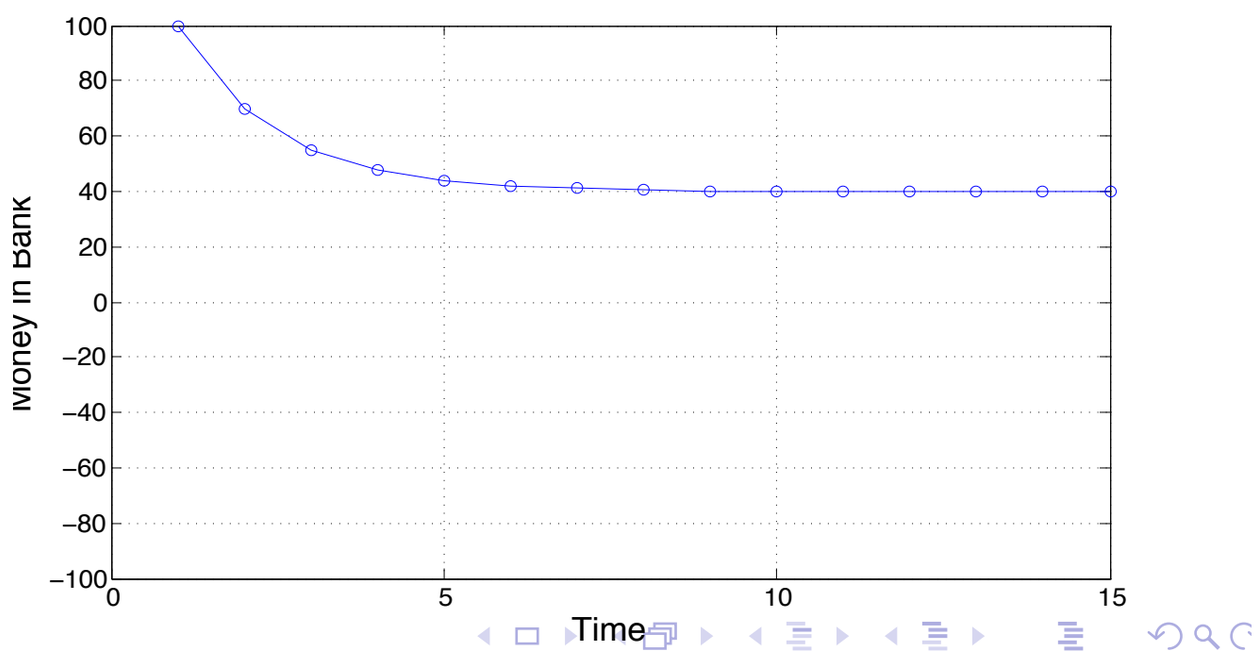


Setting the Goal

- Suppose we are trying to model a system in which the goal is not 0?
  - Keep your bank account at \$40.

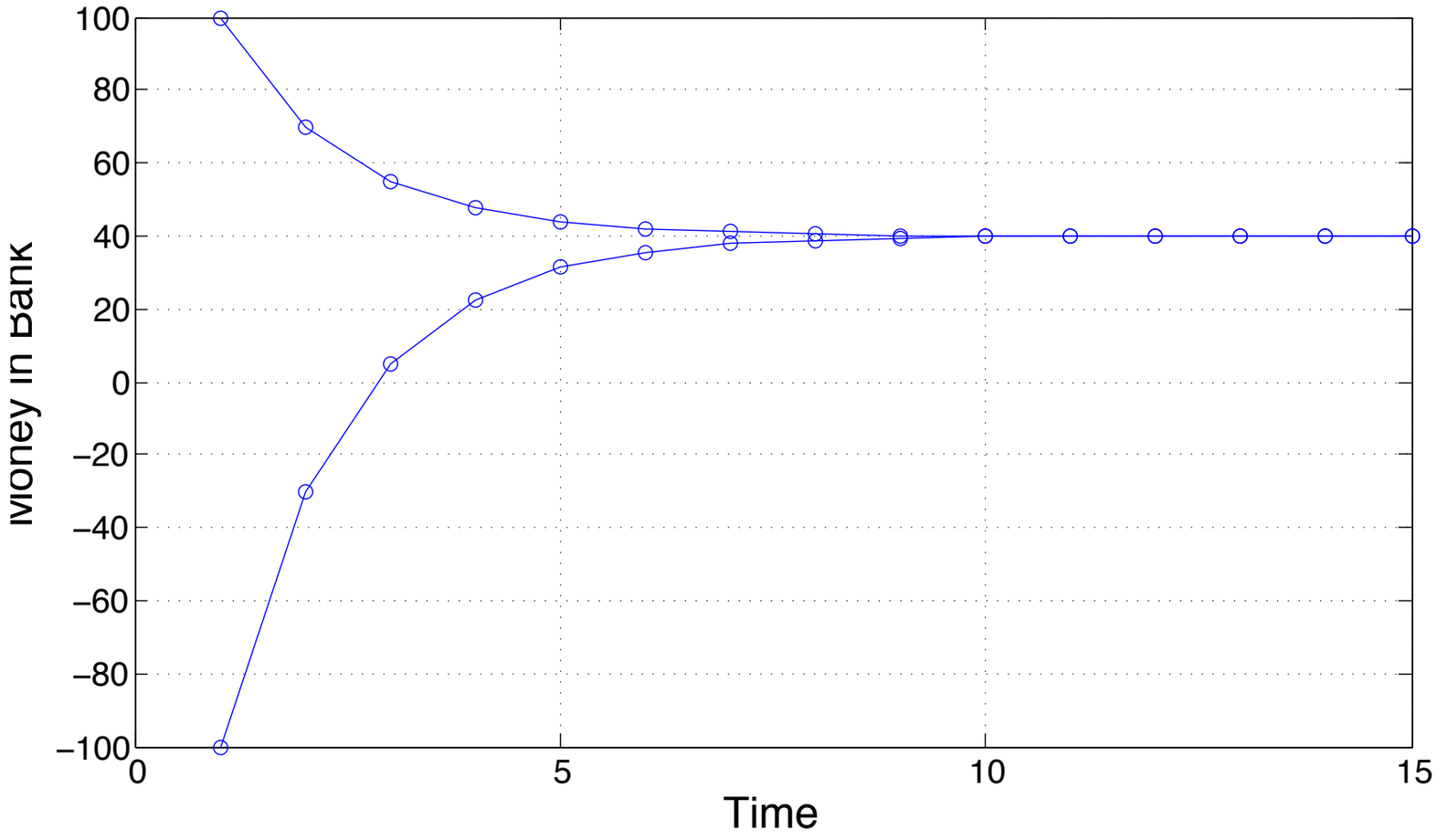
- Alternate rule for change:
  Change in x $= -\frac{1}{2}x + 20$

Time	x	change
1	100	-30
2	70	-15
3	55	-7.5
4	47.5	-3.5
5	44	-2
6	42	-1
7	41	-0.5



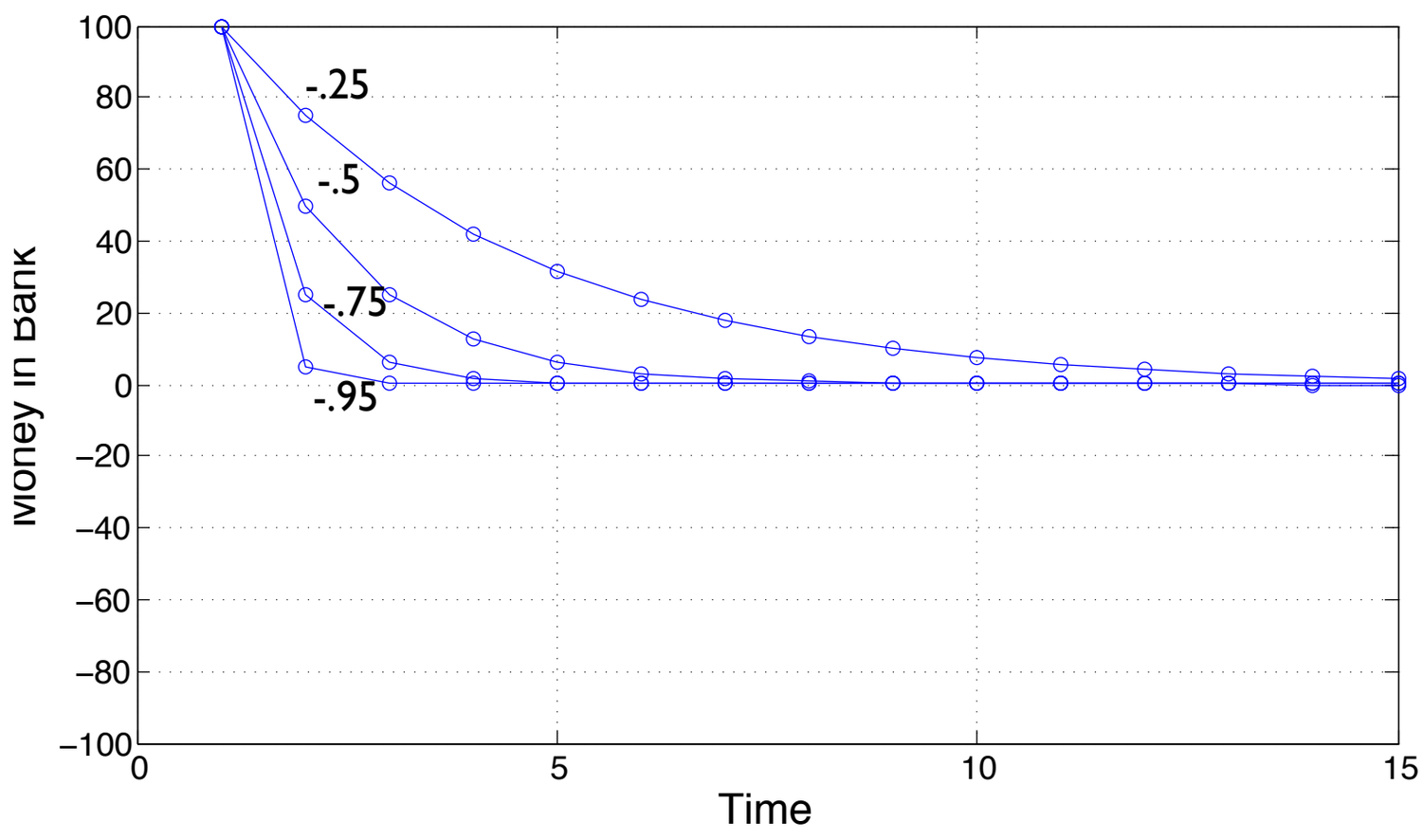
Setting the Goal

- The new goal will also be reached from any initial condition, positive or negative.
- Rule for change:
  Change in x $= -\frac{1}{2}x + 20$



Rate of Goal Attainment

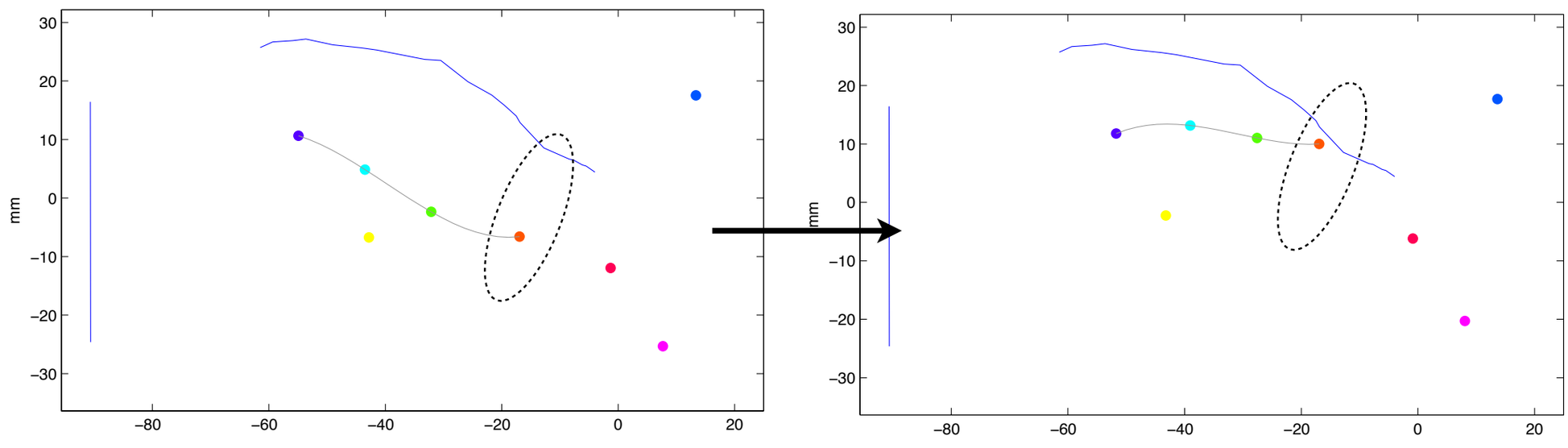
- The rate at which a system approaches its goal can be adjusted by changing the $\frac{1}{2}$ to a different fraction, e.g., .25, .75, .95.
- This parameter is sometimes referred to as k.
- Rule for change: Change in x $= -kx$
- The higher the value of k, the faster the goal is attained.



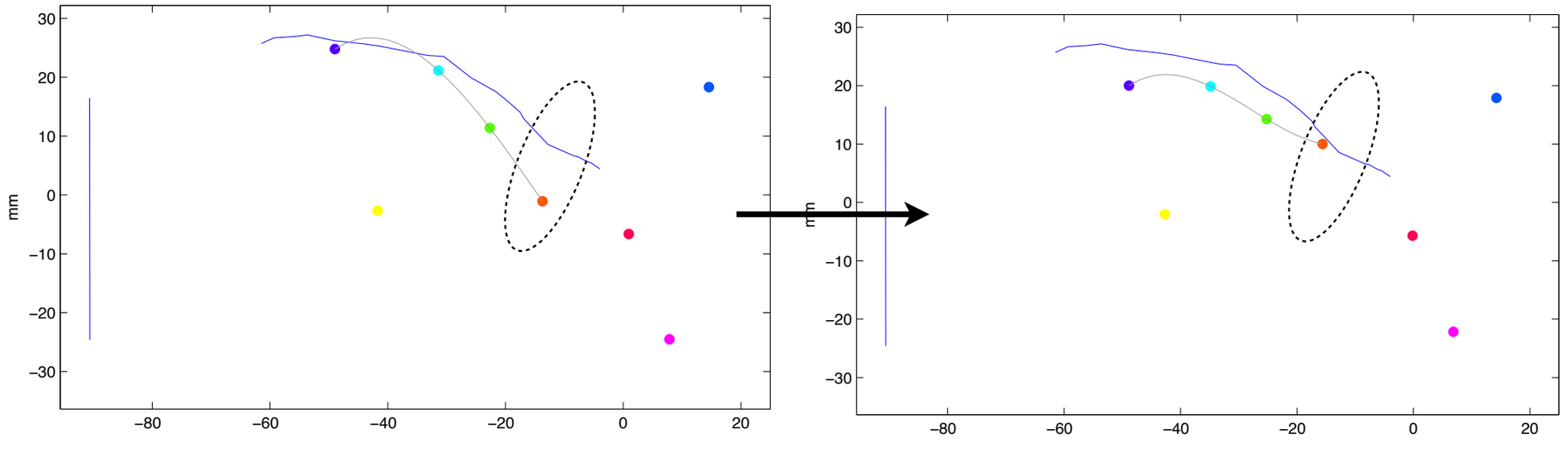
k
is also called the ***stiffness*** *of the system.*

Example of context dependence

/AA/ “sod” /D/



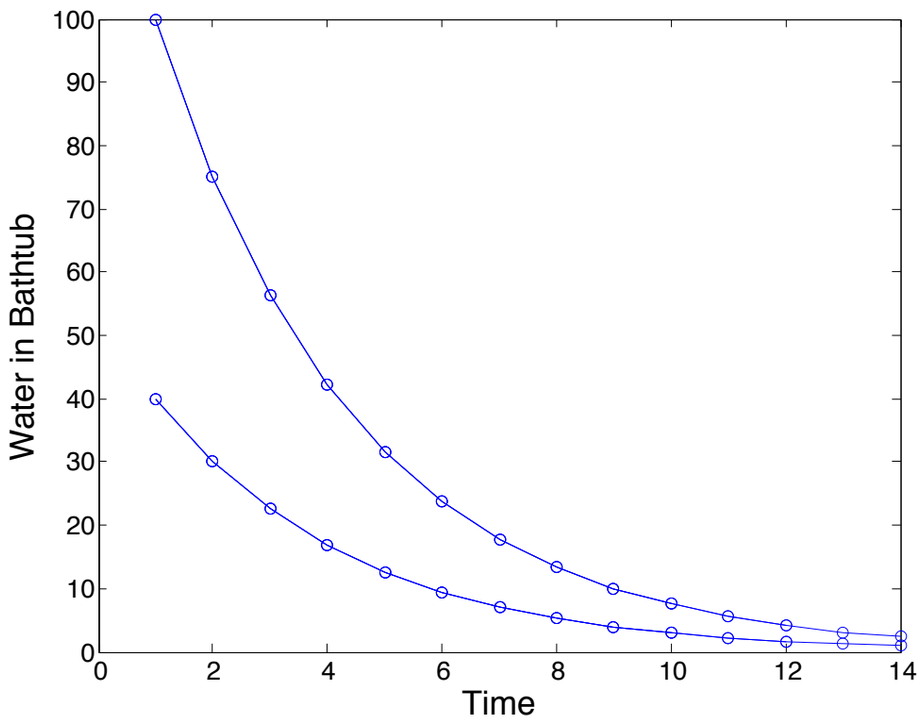
/IY/ “seed” /D/

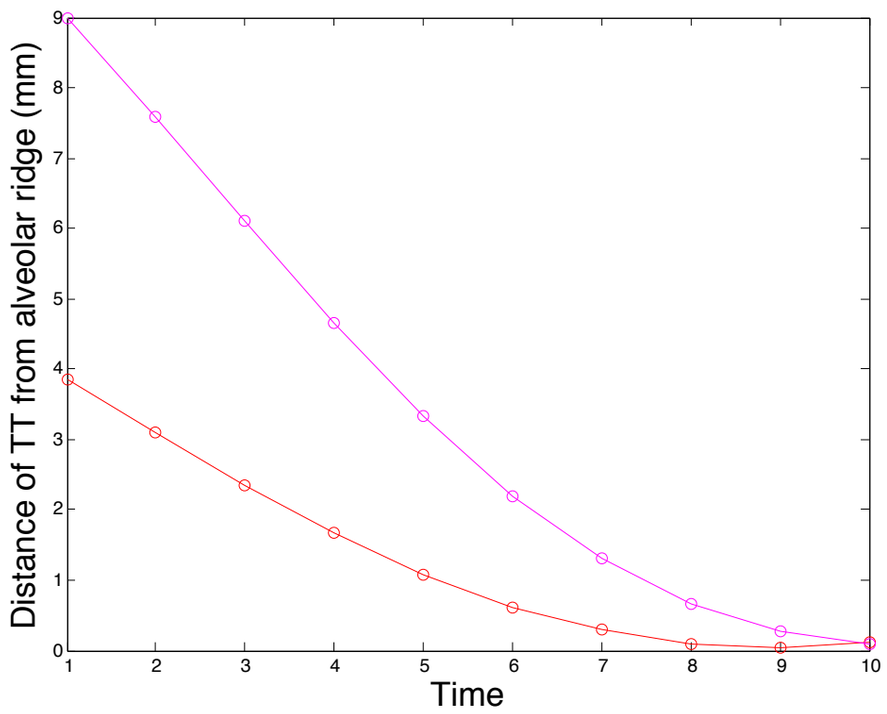


Goal-seeking and articulator motion for /d/

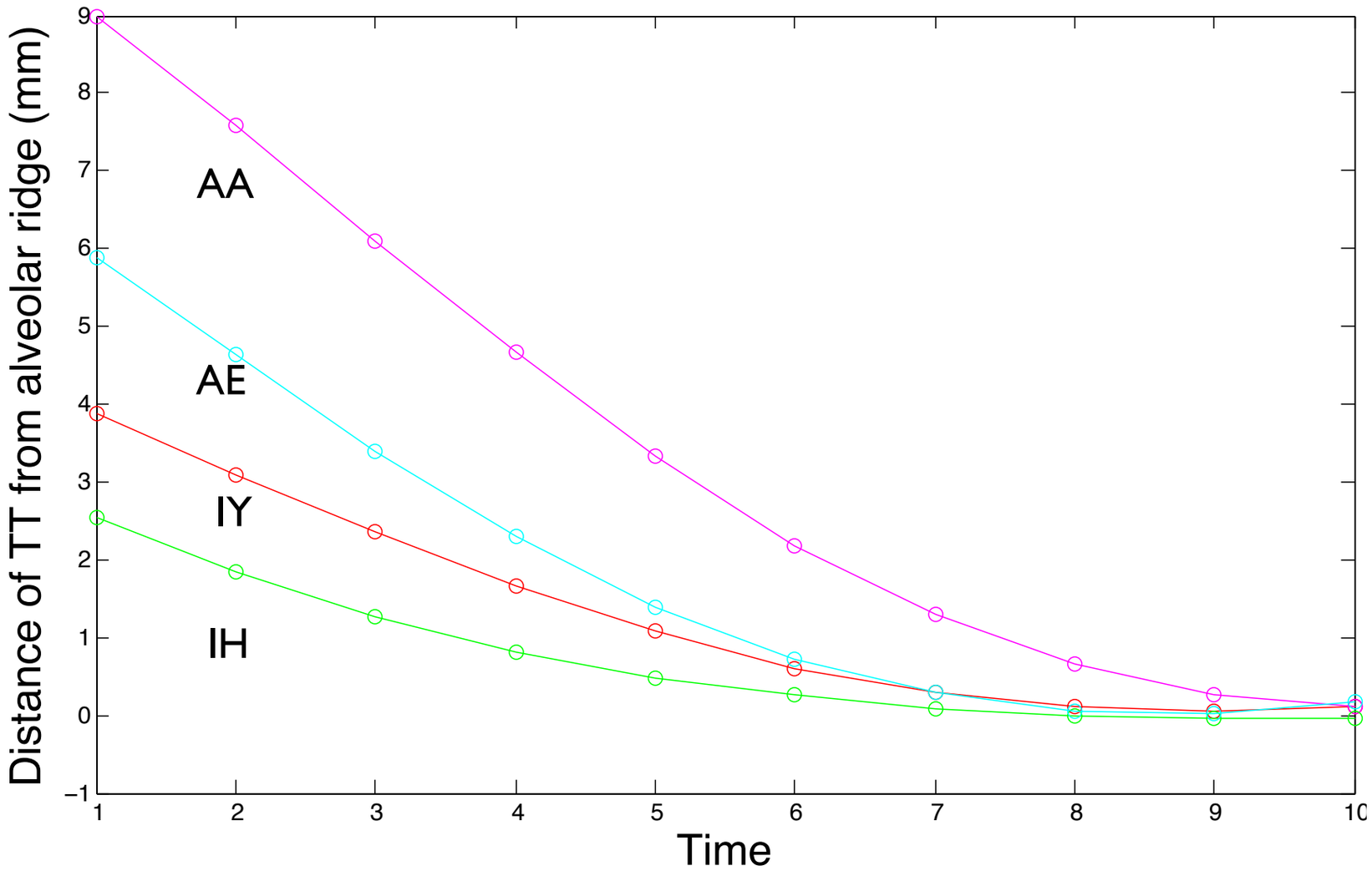
- A dynamical system with a goal of 0 can be used to model the distance of the tongue tip from the alveolar ridge over time (time-function).
- The same dynamical system will produce the different time functions we observe in "seed" and "sod".

Change in x $= -\frac{1}{4}x$





Other initial conditions

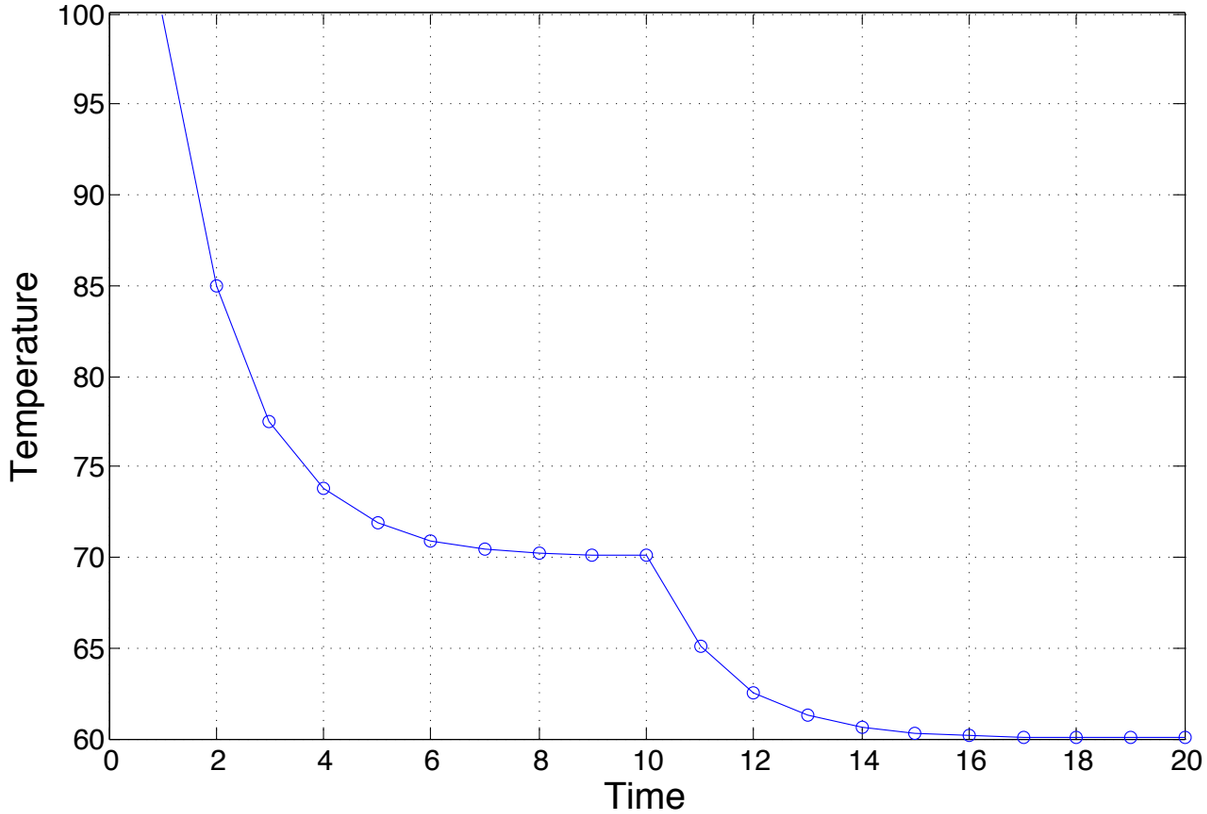


Multiple Dynamical Systems

- Possibilities
- Sequence of two goal values
- Overlap in time of two system

Sequence of Goal Values

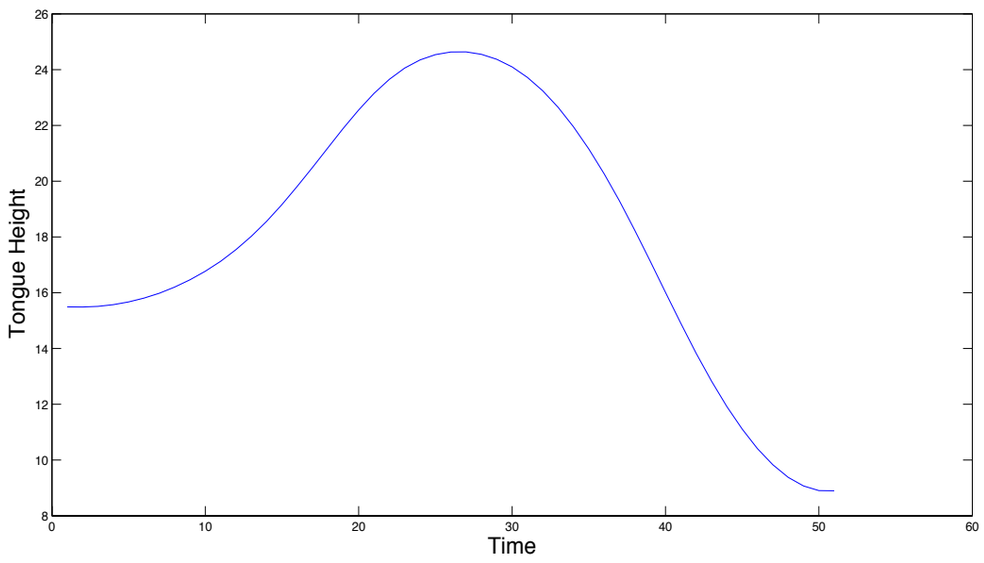
- You set the thermostat to 70, the a visitor arrives, who switches the setting to 60.
- Rule for change: Change in x $= -\frac{1}{2}x + 35$
  then Change in x $= -\frac{1}{2}x + 30$

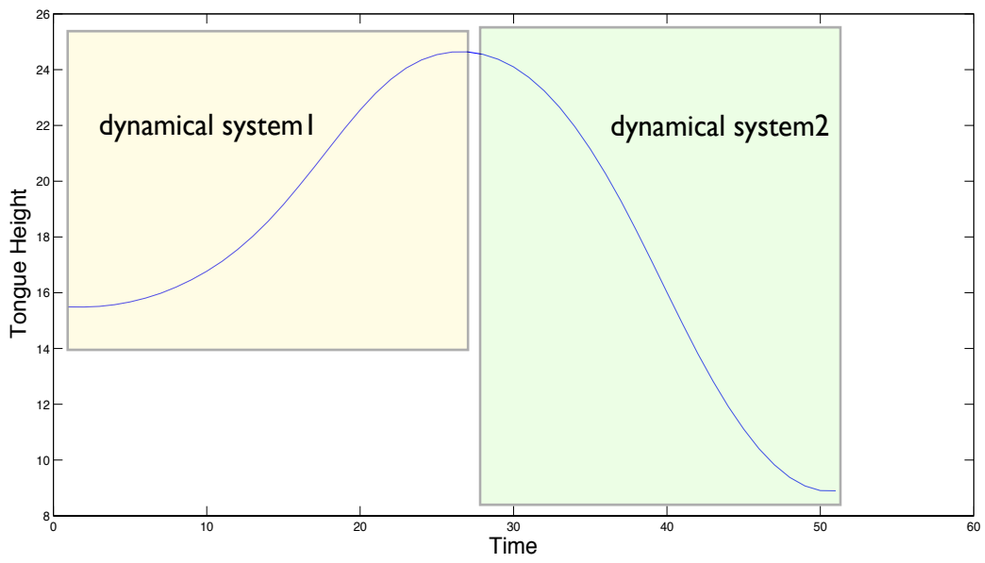


Important point:
*Dynamical systems are not forever. They are **active** and govern the behavior of a system only within some fixed epoch of time.*

Sequence: a speech example

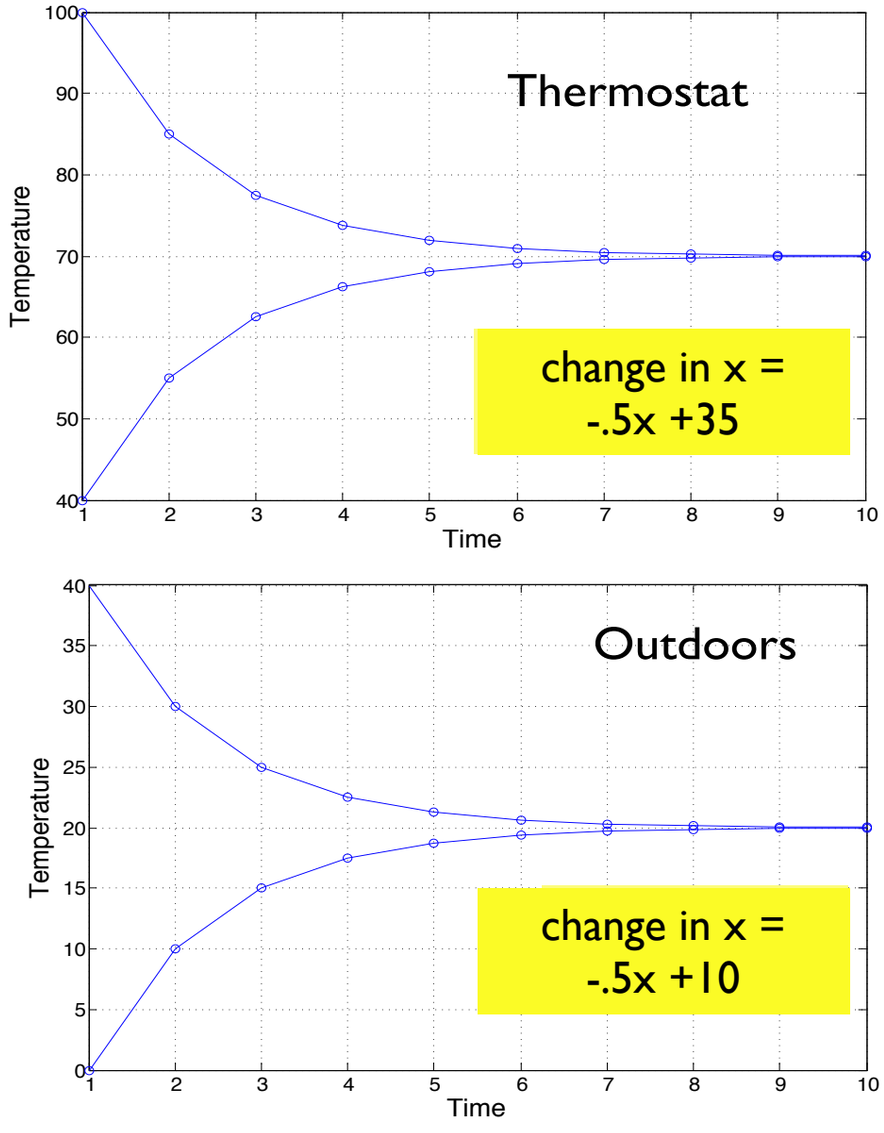
- "The other one is too big. "
- Sequence of IY (tongue body high) – AH (tongue body low)
- Model with sequence of two dynamical system Goal=25 – Goal=9;





Gestural score: *epochs during which dynamical systems of individual gestures are active (govern the vocal tract articulators).*

Overlap of two systems in time



- Imagine being in a cold climate
- Two dynamical systems:
  1. Thermostat
  2. Chill from Outside
- If the insulation is perfect, only the thermostat system will regulate inside temp.
- if there are many large open windows, only the outside system will regulate inside temp.
- If the insulation is just a bit leaky, both systems will contribute, and their effects will blend.